a review of From a geometrical point of view. A study of the history and philosophy of category theory by Marquis, Jean-Pierre

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From a geometrical point of view. A study of the history and philosophy of category theory.

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Group theory was born when the tragic mathematician Evariste Galois created the so-called Galois theory. Nowadays no one would deny the importance of group theory in mathematics and science in general, though it took almost a century for the scientific community to recognize its significance. Category theory was invented by Samuel Eilenberg and Saunders MacLane in the early forties of the preceding century, and its tremendous magnitude has been getting more and more apparent. Let alone its application to algebraic topology from which the originators of category theory got their great impetus, it was always in the background of Grothendieck's revolution in algebraic geometry in the 1950's and 1960's, and categorical ideas were also behind Faltings' proof of Mordell conjectures in the 1980's and Wiles' proof of Fermat's last theorem in the 1990's. Computer scientists understood their usefulness in the 1980's, and category theory is now a good friend of every theoretical computer scientist. It is now finding its way into mathematical physics, especially in the search for a theory of quantum gravity.

The basic tenet in this monograph is that category theory is a conceptual extension of Klein's famous program in elementary geometry in the sense that Klein's program is one very special case of the powerfulness of categorical methods. The author has tried to show that most of the basic elements implicitly and explicitly present in Klein's program transfer almost directly to category theory. The main and basic point is that category theory is thoroughly geometric. Category theory is fundamentally algebraic, and many methods developed in the study of groups have found and are finding a natural generalization in categories, in particular, presentations and representations of categories. These methods naturally lead to categories of categories, while the development of the geometric point of view in the foundations of mathematics also leads to categories of categories. In this regard, a proper characterization of the so-called weakly n-categories is to be provided, and a proper axiomatization of a category of categories as a foundation of mathematics has to be settled. These two problems still remain to be solved decisively.

The monograph consists of seven chapters. The first chapter deals with Klein's program. The second chapter looks at Eilenberg and MacLane's original claim that category theory can be considered a continuation of Klein's program, while examining their definitions of category, functor and natural transformation. Although Eilenberg and MacLane introduced categories, functors and natural transformations, they did not introduce category theory. What was missing was the algebraic structure of categories. It is a great surprise that it was not Eilenberg and MacLane but Grothendieck that introduced the correct notion of identity for categories, namely, that of the equivalence of categories. In 1957 *A. Grothendieck* published a paper [Tohoku Math. J., II. Ser. 9, 119–221 (1957; Zbl 0118.26104)] on homological algebra, where he defined abelian categories. He then used universal arrows introduced by MacLane in 1950 systematically in his revolution in algebraic geometry. *D. M. Kan* [Trans. Am. Math. Soc. 87, 294–329 (1958; Zbl 0090.38906)] introduced the concept of adjoint functors which allowed a general and unified treatment of many diverse concepts, including logical and foundational ones in general. It is now well known that universal arrows, adjoint functors and representable functors are intimately interconnected. Chapters 3–5 are devoted to the development of these notions.

No doubt, it was *F. W. Lawvere* that has launched the program of thinking the foundations of mathematics in general in a categorical framework for the first time. His Ph.D. thesis [Repr. Theory Appl. Categ. 2004, No. 5, 1–121 (2004; Zbl 1062.18004)], defended at Columbia University under Eilenberg's supervision in 1963, already contained the key ideas that have guided him throughout his career and that have influenced the categorical community so greatly. Chapter 6 starts with Lawvere's study of algebraic theories and then moves on to the category of categories and to the elementary theory of the category of sets. After these, the chapter discusses Lawvere's algebraic treatment of logic (i.e., propositional connective and quantifiers). The final section of the chapter is devoted to Ehresmann's graphical syntax, namely, sketches. The last chapter is dedicated to topos theory, a gigantic branch of category theory.

The monograph is highly readable, and it should be valuable to every expert in category theory, every novice in category theory and every mathematician who is not interested at all in category theory.

Reviewer: Hirokazu Nishimura (Tsukuba)

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